Name:______________________ Class: file

SYDNEY TECHNICAL HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE

2007

EXTENSION 1 MATHEMATICS

Instructions:

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Start each question on a new page

Total Marks – 84

- Attempt Questions 1-7
- All questions are of equal value

(For markers use only)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total

Question 1 (12 marks)

a) Find $\log_2 3$ correct to 3 decimal places. 1

b) i) Sketch $y = |2x|$ 1

ii) By drawing suitable lines on your sketch above, determine that one of the following equations $A : |2x| = x - 1$ and $B : |2x| = 1 - x$ has no solutions and solve the other. 3

c) Find $\lim_{x \to 0} \frac{\sin 2x}{\frac{1}{2}x}$ 1

d) If α, β and δ are the roots of $2x^3 + 12x^2 - 6x + 1 = 0$ find the values of

i) $\alpha + \beta + \delta$ 1

ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\delta}$ 2

e) Use the substitution $u = 4 - x^2$ or otherwise to find $\int x\sqrt{4 - x^2}\,dx$ 3

Question 2 (12 marks) (start a new page)

a) Given that $\log_x 2 = a$ and $\log_x 3 = b$ find $\log_x 2.25$ in terms of a and b. 2

b) Evaluate $\int_{-1}^{2} |1 - 2x|\,dx$ by considering a graph or otherwise. 2

c) Find i) $\int \frac{3x}{x^2 + 1}\,dx$ 2

ii) $\int \frac{3}{x^2 + 1}\,dx$ 2

d) Solve $\sin 2\theta = \sin\theta$ for $0 \le \theta \le 2\pi$ 2

e) An area of 1 unit^2 is bounded by the curve $y = \frac{1}{x}$, the x axis and the lines $x = e$ and $x = k$ 2

Find the value of k (in exact form), if $k > e$.

Question 3 (12 marks) (start a new page)

a) Find $\int \cos^2 3x \; dx$ 2

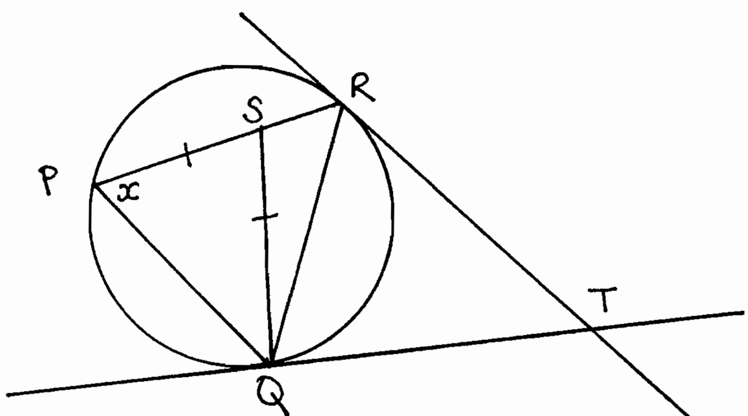
b) i) Show that $\tan 75° = \sqrt{3} + 2$ 2

ii) The lines $y = mx$ and $x = y\sqrt{3}$ meet at an angle of $75°$. Find only one value of m. 2

c) PQR is a triangle inscribed in a circle. S is a point on PR, chosen so that $QS = SP$. Tangents from an external point T touch the circle at Q and R.

Copy the diagram onto your page and prove that the quadrilateral $QTRS$ is cyclic.

Let $\angle SPQ = x$



3

d) i) Show that $\frac{d}{dx}\left[\tan^{-1}(e^x) + \tan^{-1}(e^{-x})\right] = 0$ 2

ii) Hence evaluate $\tan^{-1}(e^x) + \tan^{-1}(e^{-x})$ for all values of x. 1

Question 4 (12 marks) (start a new page)

a) If $y = xe^x$

i) Prove $\dfrac{dy}{dx} = e^x(x+1)$ and $\dfrac{d^2y}{dx^2} = e^x(x+2)$ 2

ii) Hence prove by mathematical induction for all positive integers n, that

$\dfrac{d^n y}{dx^n} = e^x(x+n)$ 3

b) For the curve $y = 2\sin^{-1}(1-4x)$, state the domain and range and sketch the graph. 3

c) The point $P(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$. The line ℓ is a tangent at P

i) Write the equation of ℓ 1

ii) If ℓ meets the y axis at A, show that SP=SA where S *is* the focus of the parabola. 2

iii) Hence show that ℓ is equally inclined to SP and the axis of the parabola. 1

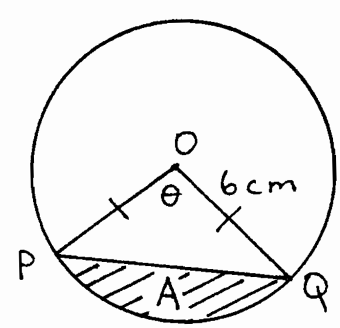
Question 5 (12 marks) (start a new page)

a) i) The polynomial equation $P(x) = 0$ has a double root at $x = a$. By putting $P(x) = (x-a)^2.Q(x)$ show that $P'(a) = 0$ 2

ii) You are told the polynomial $P(x) = mx^4 + nx^3 - 6x^2 + 22x - 12$ has a double root at $x = 1$. Find the value of m and n. 3

b) O is the centre of a circle with radius 6cm.

$\angle POQ = \theta$ radians



i) Find an expression for A, the area of the minor segment, cut off by the chord PQ, in terms of θ. 1

ii) If θ is increasing at 0.75 radians/second, what is the rate of change of A when $\theta = \frac{\pi}{3}$? 2

c) Katrina, a sky-diver, opens her parachute when falling at 30m/s. Thereafter her acceleration is given by $\frac{dv}{dt} = k(6 - v)$ where k is a constant.

i) Show that this condition is satisfied when $v = 6 + Ae^{-kt}$ and find the constant A. 2

ii) One second after opening her chute, her velocity has fallen to 10.7 m/s. Find the value of k correct to 2 decimal places. 1

iii) Find her velocity, correct to 1 decimal place, two seconds after her chute has opened. 1

Question 6 (12 marks) (start a new page)

a) i) Show that $\frac{d}{dx}(\frac{1}{2}v^2) = \frac{d^2x}{dt^2}$ 1

ii) An object is falling through a fluid in such a way that its acceleration is given by, $\frac{d^2x}{dt^2} = \frac{4}{\sqrt{x}}$ were x is the distance the object has fallen in metres and t is time in seconds. 3.

If the object started from rest, how fast would it be travelling after falling through a distance of 7 metres. (to 1 decimal place)?

b) i) Sketch the function $f(x) = x + \frac{1}{x}$ for $x > 0$ showing the stationary point and asymptotes. 2

ii) State the largest possible domain containing $x = 2$ for which $f(x)$ has an inverse $f^{-1}(x)$. 1

iii) Sketch $y = f^{-1}(x)$ on the diagram above. 1

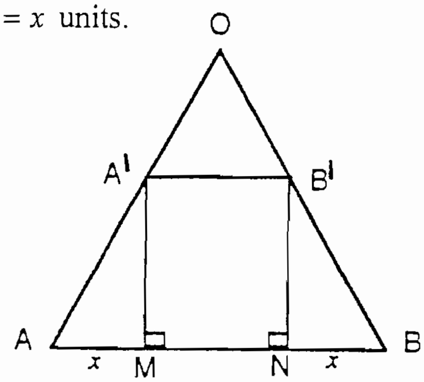
iv) Show that $f^{-1}(x) = \frac{x}{2} + \frac{1}{2}\sqrt{x^2 - 4}$ 2

v) Assume $x = N$, when N is not in the domain chosen for part ii) but still in the domain for $f(x)$.

Find $f^{-1}[f(N)]$ 2

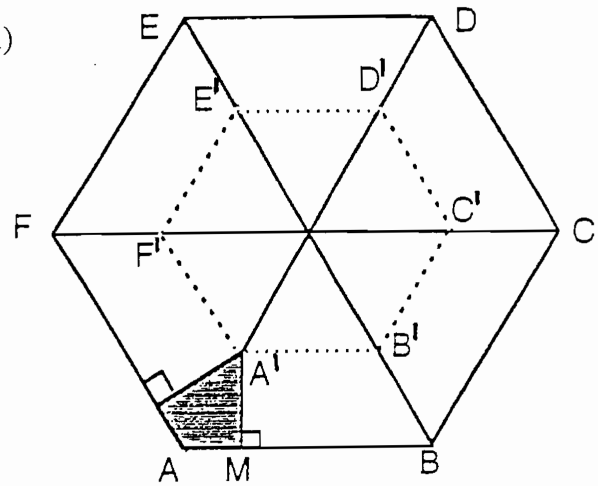
Question 7 **(12 marks)** **(start a new page)**

a) i) OAB is an equilateral triangle side m units. $A^1B^1 \parallel AB$ and $AM = NB = x$ units.



Show that the area of ΔOA^1B^1 is given by $\dfrac{\sqrt{3}(m-2x)^2}{4}$ 2

ii)



Use part i) as you answer part ii)

$ABCDEF$ is a regular hexagon, side m units. The sides of $A^1B^1C^1D^1E^1F^1$ are parallel to those of $ABCDEF$. From each vertex, portions such as the one shaded are removed. The remainder is folded along the dotted lines to form a hexagonal prism

α) If $AM = x$ units prove the volume of the prism is given by

$$V = \frac{9x(m-2x)^2}{2} \text{ units}^3$$ 2

β) Prove that the maximum volume of such a prism is $\dfrac{m^3}{3}$ units^3 3

b) If $\tan 2x = \dfrac{\tan x}{a\tan x + b}$ and $\tan x \neq 0$

i) Find a condition in terms of a and b, for the equation above, to have two different roots $\tan\alpha$ and $\tan\beta$ 2

ii) Assuming this condition to be satisfied prove $\tan^2(\alpha-\beta) = \dfrac{a^2-2b+1}{}$ 3

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1

a)

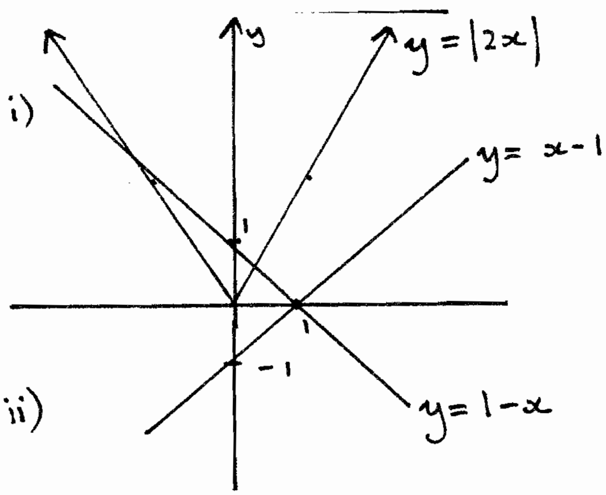
$$\log_2 3 = \frac{\log_e 3}{\log_e 2}$$

$$= \underline{\underline{1.585}} \text{ (3 dec. pl.)}$$

b)

i)

ii)



$\therefore |2x| = x - 1$ no solutions; no pts of intersection

$|2x| = 1 - x$ 2 solutions

$2x = 1 - x \qquad 2x = -(1 - x)$

$\underline{\underline{x = \frac{1}{3} \quad \text{and} \quad x = -1}}$

c) $\displaystyle \lim_{x \to 0} \frac{\sin 2x}{\frac{1}{2}x}$

$= \lim \quad \sin 2x \quad \frac{2}{}$

d) $a = 2 \quad b = 12 \quad c = -6 \quad d = 1$

i) $\alpha + \beta + \delta = \frac{-b}{a} = \frac{-12}{2} = \underline{\underline{-6}}$

ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\delta} = \frac{\beta\delta + \alpha\delta + \alpha\beta}{\alpha\beta\delta}$

$$= \frac{\frac{c}{a}}{-\frac{d}{a}} = \frac{-3}{-1/2}$$

$$\underline{\underline{= 6}}$$

e) $u = 4 - x^2$

$\frac{du}{dx} = -2x$

$du = -2x \, dx$

$\therefore dx = \frac{du}{-2x}$

$$\int x\sqrt{4 - x^2}\, dx = \int \cancel{x} . \sqrt{u} . \frac{du}{-2\cancel{x}}$$

$$= -\frac{1}{2}\int u^{1/2}\, du$$

$$= -\frac{1}{2}\left[\frac{2u^{3/2}}{3}\right] + c$$

$$\underline{\underline{= -\frac{1}{3}\sqrt{(4 - x^2)^3} + c}}$$

Question 2

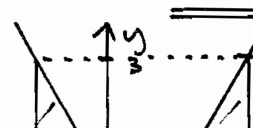
a) $\log_x 2.25 = \log_x \frac{9}{4}$

$= \log_x 9 - \log_x 4$

$= 2\log_x 3 - 2\log_x 2$

$\underline{\underline{= 2b - 2a}}$

b)



$\therefore \int^2 |1 - 2x|\, dx$

c) i) $\int \frac{3x}{x^2+1}\,dx = \frac{3}{2}\int \frac{2x}{x^2+1}\,dx$

$= \frac{3}{2}\ln(x^2+1) + c$

ii) $\int \frac{3}{x^2+1}\,dx = 3\int \frac{1}{x^2+1}\,dx$

$= 3\tan^{-1}x + c$

d) $\sin 2\theta = \sin\theta$

$2\sin\theta.\cos\theta = \sin\theta$

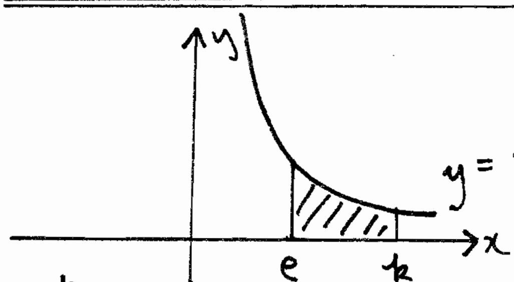
$2\sin\theta.\cos\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0 \qquad \cos\theta = \frac{1}{2}$

$\theta = 0, \pi, 2\pi \qquad \theta = \frac{\pi}{3}, \frac{5\pi}{3}$

e)



$\int_e^k \frac{1}{x}\,dx = 1$

$[\ln x]_e^k = 1$

$\ln k - \ln e = 1$

$\ln k - 1 = 1$

$\log_e k = 2$

$k = e^2$

Question 3

$\therefore \int \cos^2 3x\,dx = \frac{1}{2}\int(\cos 6x + 1)\,dx$

$= \frac{1}{2}\left[\frac{1}{6}\sin 6x + x\right] + c$

$= \frac{1}{12}\sin 6x + \frac{x}{2} + c$

b) i) $\tan 75^\circ = \tan(30+45)^\circ$

$= \frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ.\tan 45^\circ}$

$= \left(\frac{1}{\sqrt{3}} + 1\right) \div \left(1 - \frac{1}{\sqrt{3}}\right)$

$= \frac{1+\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}-1}$

$= \frac{1+\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{2\sqrt{3}+4}{2}$

$= \sqrt{3} + 2$

ii) gradients are m and $\frac{1}{\sqrt{3}}$

$\tan 75 = \frac{m - \frac{1}{\sqrt{3}}}{1 + \frac{m}{\sqrt{3}}}$ (take +ve case only one solution required)

$\sqrt{3} + 2 = \frac{m - \frac{1}{\sqrt{3}}}{1 + \frac{m}{\sqrt{3}}}$

$(\sqrt{3}+2)\left(1 + \frac{m}{\sqrt{3}}\right) = m - \frac{1}{\sqrt{3}}$

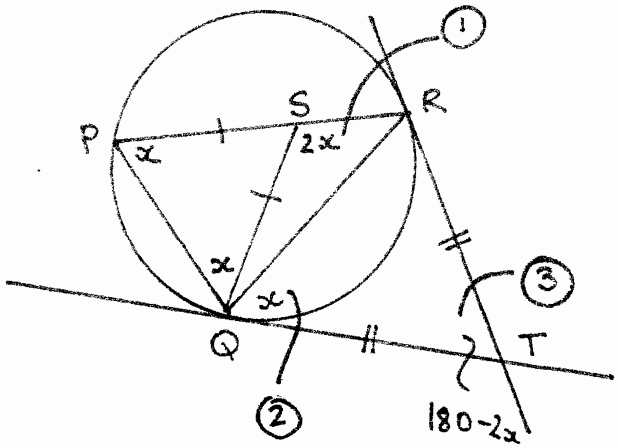
$\sqrt{3} + m + 2 + \frac{2m}{\sqrt{3}} = m - \frac{1}{\sqrt{3}}$

$\frac{2m}{\sqrt{3}} = -\frac{1}{\sqrt{3}} - \sqrt{3} - 2$

$2m = -1 - 3 - 2\sqrt{3}$

$m = -2 - \sqrt{3}$

c)



$R\hat{S}Q = 2x$ (ext. angle of isosceles triangle)

$R\hat{Q}T = x$ (alt. segment theorem)

since $QT = TR$ (tangents from a external pt are =)

$R\hat{T}Q = 180 - 2x$ (angle sum of isosceles triangle)

$R\hat{S}Q + R\hat{T}Q = 180^\circ$

$\therefore$ QTRS is cyclic since opposite angles are supp.

d) i) $\frac{d}{dx}\left(\tan^{-1} e^{x} + \tan^{-1} e^{-x}\right)$

$$= \frac{e^{x}}{e^{2x}+1} - \frac{e^{-x}}{e^{-2x}+1}$$

$$= \frac{e^{x}}{e^{2x}+1} - \left[\frac{1}{e^{x}} \div \left(\frac{1}{e^{2x}}+1\right)\right]$$

$$= \frac{e^{x}}{e^{2x}+1} - \left[\frac{1}{\cancel{e^{x}}} \times \frac{\cancel{e^{2x}}^{\,e^{x}}}{1+e^{2x}}\right]$$

$$= e^{x} - e^{x}$$

ii) subst. $x = 0$ since true for all x

$\tan^{-1}(e^{0}) + \tan^{-1}(e^{0})$

$2\tan^{-1} 1 = 2 \times \frac{\pi}{4}$

$= \pi/2$

Question 4

a) $y = xe^{x}$ | $u = x$ $v = e^{x}$; $u' = 1$ $v' = e^{x}$

i) $\frac{dy}{dx} = e^{x} + xe^{x}$

$\frac{dy}{dx} = e^{x}(1+x)$

$\frac{d^2y}{dx^2} = e^{x} + e^{x}(1+x)$

$= e^{x}(1+1+x)$

$\therefore \frac{d^2y}{dx^2} = e^{x}(2+x)$

ii) Step 1: Show true for $n = 1$

$\therefore \frac{dy}{dx} = e^{x}(x+1)$ from above

Step 2: Assume true for $n = k$ some +ve integer

$\frac{d^{k}y}{dx^{k}} = e^{x}(x+k)$

Step 3: Prove true for $n = k+1$

$\left\{ \text{ie show that } \frac{d^{(k+1)}y}{dx^{(k+1)}} = e^{x}(x+k+1) \right\}$

LHS $= \frac{d^{(k+1)}y}{dx^{(k+1)}}$

$= \frac{d}{dx}\left(e^{x}(x+k)\right)$ from Step 2

$= \frac{d}{dx}\left(x.e^{x} + k e^{x}\right)$

$= e^{x} + x e^{x} + k e^{x}$

$= e^{x}(1 + x + k)$

$=$ RHS

Step 4: Since true for $n=1$ and if assumed true for $n=k$ (some +ve integer) we have shown true for $n=k+1$

$\therefore$ true for all +ve integers

b) $y = 2\sin^{-1}(1-4x)$

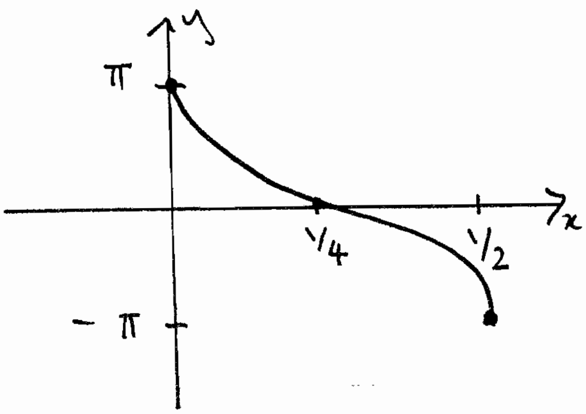
$\frac{y}{2} = \sin^{-1}(1-4x)$

$-\pi/2 \leq \frac{y}{2} \leq \pi/2$

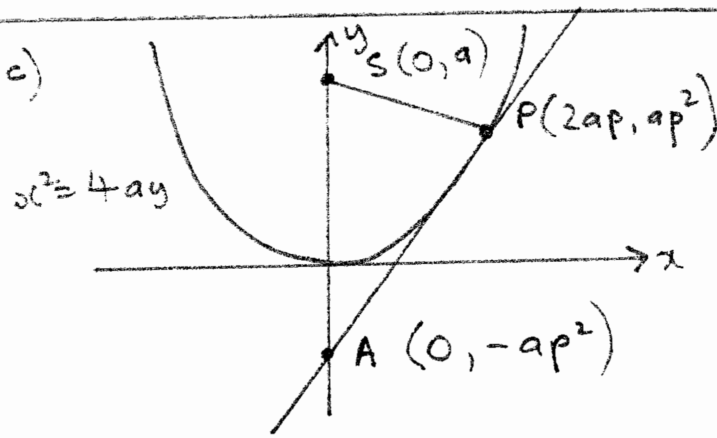
$\therefore$ Range : $-\pi \leq y \leq \pi$

$-1 \leq 1-4x \leq 1$

Domain : $0 \leq x \leq 1/2$



c)



i) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a} = \frac{x}{2a}$

$m_p = \frac{2ap}{2a} = p$

tang: $y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap$

$y = px - ap^2$

ii) $A(0, -ap^2)$

$SA = a + ap^2 = a(1+p^2)$

$SP = \sqrt{(2ap-0)^2 + (ap^2-a)^2}$

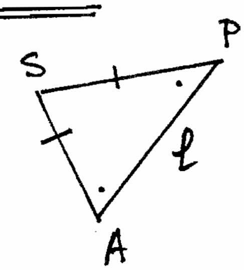
$= \sqrt{4a^2p^2 + a^2p^4 - 2a^2p^2 + a^2}$

$= a\sqrt{p^4 + 2p^2 + 1}$

$= a\sqrt{(p^2+1)^2}$

$SP = a(p^2+1)$

iii) $\therefore SP = SA$



ΔSPA is isosceles

$\therefore S\hat{A}P = S\hat{P}A$

Question 5

a) i) $P(x) = (x-a)^2 . Q(x)$

$u = (x-a)^2 \qquad v = Q(x)$

$u' = 2(x-a) \qquad v' = Q'(x)$

$P'(x) = 2(x-a).Q(x) + (x-a)^2.Q'(x)$

$P'(a) = 2(a-\cancel{a})Q(a) + (a-\cancel{a})^2.Q'(\ )$

$\therefore P'(a) = 0$

ii) $P(1) = 0$ and $P'(1) = 0$

$P(x) = mx^4 + nx^3 - 6x^2 + 22x - 12$

$P(1) = m + n - 6 + 22 - 12 = 0$

$m + n = -4$ —— ①

$P'(x) = 4mx^3 + 3nx^2 - 12x + 22$

$P'(1) = 4m + 3n - 12 + 22 = 0$

$4m + 3n = -10$ —— ②

① × 4 : $4m + 4m - 16$

② $4m + 3n = -10$

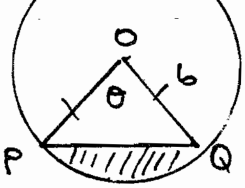
$n = -6$

$m - 6 = -4$

$m = 2$

$\therefore m = 2,\ n = -6$

b)



i) $A = \frac{1}{2}.6^2(\theta - \sin\theta)$

$A = 18\theta - 18\sin\theta$

ii) $\frac{d\theta}{dt} = .75$ require $\frac{dA}{dt}$ when $\theta = \pi/2$

$\frac{dA}{d\theta} = 18 - 18\cos\theta$

$\therefore \frac{dA}{dt} = .75\left(18 - 18\cos\frac{\pi}{3}\right)$

$= .75\left(18 - 18.\frac{1}{2}\right)$

$\frac{dA}{dt} = 6.75\ cm^2/\text{second}$

c) i) $V = 6 + Ae^{-kt}$

$\therefore \frac{dV}{dt} = -k(Ae^{-kt})$

$= -k(V-6)$

$= -kV + 6k$

$\therefore \frac{dV}{dt} = k(6-V)$

as required

$V = 30\ m/s \quad t = 0$ subs.

$\therefore 30 = 6 + Ae^0$

$\therefore A = 24$

ii) $t = 1 \quad V = 10.7$

$V = 6 + 24e^{-kt}$

$10.7 = 6 + 24e^{-k}$

$4.7 = 24e^{-k}$

$\ln\left(\frac{4.7}{24}\right) = -k$

$k = 1.63$ (2 dec. pl)

iii) $t = 2$

$V = 6 + 24e^{-1.63\times 2}$

$V = 6.9\ m/s$ (1 dec. pl)

Question 6

a) i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dv}\left(\frac{1}{2}v^2\right)\cdot\frac{dv}{dx}$

$$= v\cdot\frac{dv}{dx}$$

$$= \frac{\cancel{dx}}{dt}\cdot\frac{dv}{\cancel{dx}}$$

$$= \frac{dv}{dt}$$

$$= \ddot{x}$$

ii) $\ddot{x} = \frac{4}{\sqrt{x}}$

$t=0,\ \dot{x}=0,\ x=0$

$$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = 4x^{-1/2}$$

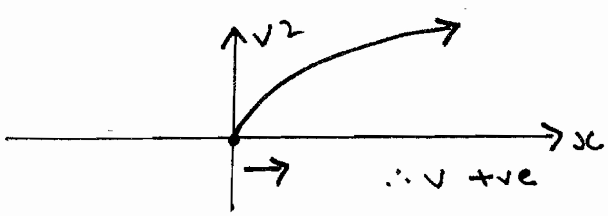
$$\frac{1}{2}v^2 = \frac{4x^{1/2}}{1/2} + c_1$$

$$= 8\sqrt{x} + c_1$$

$$v^2 = 16\sqrt{x} + c$$

sub $x=0$ $v=0$ $\therefore c=0$

$$v^2 = 16\sqrt{x}$$



$\therefore v$ +ve

sub. $x=7$

$$v^2 = 16\sqrt{7}$$

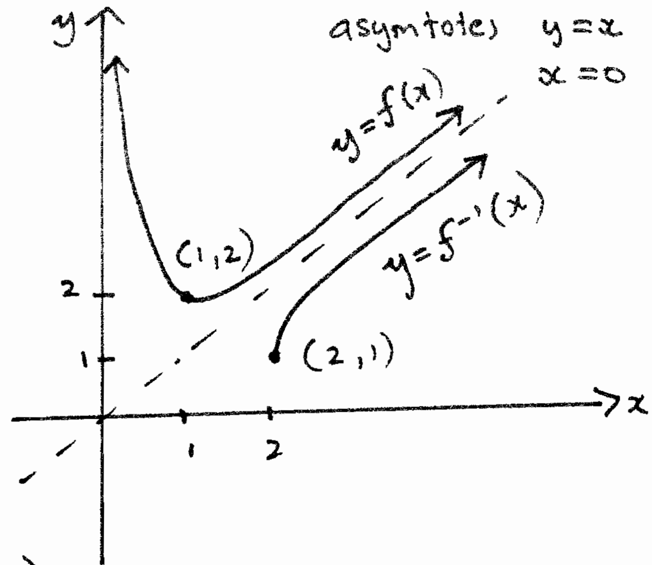
$$v = 6.5\text{ m/s}$$

b) i) $f(x) = x + x^{-1}$

$f'(x) = 1 - x^{-2}$

since $x>0$ at $(1,2)$ $f''(x)>0$ min

asymtotes $y=x$ &
$x=0$



ii) D: $x \geq 1$

iii) see diagram

iv) $x = y + \frac{1}{y}$

$$xy = y^2 + 1$$

$$-1 = y^2 - xy$$

$$-4 = 4y^2 - 4xy$$

$$-4 + x^2 = 4y^2 - 4xy + x^2$$

$$-4 + x^2 = (2y - x)^2$$

$$\pm\sqrt{x^2 - 4} = 2y - x$$

from domain above take $+\sqrt{\ }$

$$2y = x + \sqrt{x^2 - 4}$$

$$\therefore y = \frac{x}{2} + \frac{1}{2}\sqrt{x^2 - 4}$$

v) $f^{-1}[f(N)]$

N not in above domain

$\therefore$ use $f^{-1}(x) = \frac{x}{2} - \frac{1}{2}\sqrt{x^2-4}$

$f(N) = N + \frac{1}{N}$

$$f^{-1}(f(N)) = \frac{1}{2}\left(N+\frac{1}{N}\right) - \frac{1}{2}\sqrt{\left(N+\frac{1}{N}\right)^2 - 4}$$

$$= \frac{N}{2} + \frac{1}{2N} - \frac{1}{2}\sqrt{N^2+2+\frac{1}{N^2}-4}$$

$$= \frac{N}{2} + \frac{1}{2N} - \frac{1}{2}\sqrt{N^2-2+\frac{1}{N^2}}$$

N.B. maintain the -ve here

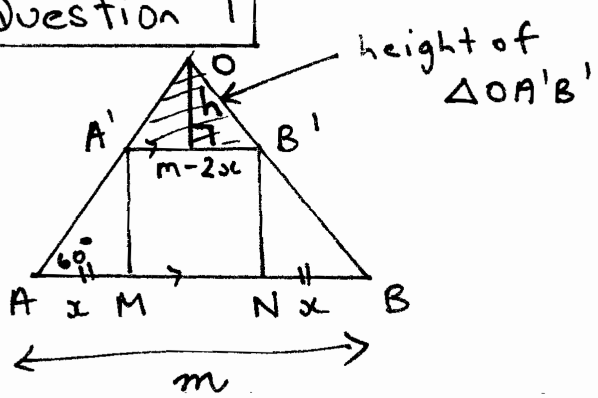
$$= \frac{N}{2} + \frac{1}{2N} - \frac{1}{2}\sqrt{\left(N-\frac{1}{N}\right)^2}$$

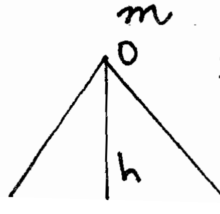
$$= \frac{N}{2} + \frac{1}{2N} - \frac{1}{2}\left(N-\frac{1}{N}\right)$$

$$= \cancel{\frac{N}{2}} + \frac{1}{2N} - \cancel{\frac{N}{2}} + \frac{1}{2N}$$

$$f^{-1}(f(N)) = \frac{1}{N}$$

Question 7





In Δ OA'B'

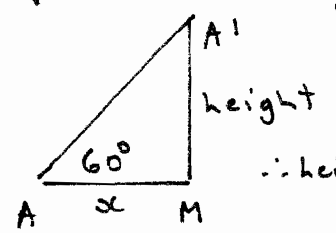
$$\tan 60^\circ = \frac{h}{\frac{m-2x}{2}}$$

$$\frac{\sqrt{3}(m-2x)}{2} = h$$

$\therefore$ Area of OA'B' $= \frac{1}{2}(m-2x)\frac{\sqrt{3}}{2}(m-2x)$

$$A = \frac{\sqrt{3}}{4}(m-2x)^2$$

ii) V = Abase × height



$\therefore$ height $= x\tan 60^\circ$

$= x\sqrt{3}$

$$V = 6\left(\frac{\sqrt{3}}{4}(m-2x)^2\right).\ x\sqrt{3}$$

$$V = \frac{9}{2}x(m-2x)^2$$

iii) α) $u = \frac{9x}{2}$ $\quad v = (m-2x)^2$

$u' = \frac{9}{2}$ $\quad v' = 2 . -2(m-2x)$

$v' = -4(m-2x)$

using product rule

$$V' = \frac{9}{2}(m-2x)^2 - 18x(m-2x)$$

$$V' = 9(m-2x)\left[\frac{1}{2}(m-2x) - 2x\right]$$

$$V' = 9(m-2x)\left(\frac{m}{2} - 3x\right)$$

st pt $V' = 0$

$\therefore$ $x = \frac{m}{2}$ OR $x = \frac{m}{6}$

test max/min

x	$m/8$	$m/6$	$m/4$	$m/2$	m
V'	+	0	−	0	+

+ / max \ − − \ 0 / +

β) sub $x = \frac{m}{6}$ into V

$$V = \frac{9}{2} \cdot \frac{m}{6} \cdot \left(m - 2\frac{m}{6}\right)^2$$

$$= \frac{3m}{4}\left(\frac{2m}{3}\right)^2$$

$$= \frac{3m}{4} \cdot \frac{4m^2}{9}$$

$$\text{max. } V = \underline{\underline{\frac{m^3}{3} \text{ unit}^3}}$$

b) i) different roots if $\Delta > 0$

$$\tan 2x = \frac{\tan x}{a\tan x + b}$$

$$\frac{2\tan x}{1 - \tan^2 x} = \frac{\tan x}{a\tan x + b}$$

$$2\tan x\,(a\tan x + b) = \tan x\,(1 - \tan^2 x)$$

since $\tan x \neq 0$ given

$$2(a\tan x + b) = 1 - \tan^2 x$$

$$2a\tan x + 2b = 1 - \tan^2 x$$

$$\tan^2 x + 2a\tan x + 2b - 1 = 0$$

$$\Delta = (2a)^2 - 4 \cdot 1 \cdot (2b-1)$$

require $\Delta > 0$

$$4a^2 - 8b + 4 > 0$$

$$\underline{\underline{a^2 - 2b + 1 > 0}}$$

ii) $\text{LHS} = \tan^2(\alpha - \beta)$

$$= \left[\tan(\alpha - \beta)\right]^2$$

$$= \left(\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \cdot \tan\beta}\right)^2$$

$$= \frac{\tan^2\alpha + \tan^2\beta - 2\tan\alpha \cdot \tan\beta}{(1 + \tan\alpha \cdot \tan\beta)^2}$$

(since $A^2 + B^2 = (A+B)^2 - 2AB$)

$$= \frac{(\tan\alpha + \tan\beta)^2 - 4\tan\alpha \cdot \tan\beta}{(1 + \tan\alpha \cdot \tan\beta)^2}$$

$$= \frac{(\text{sum of roots})^2 - 4 \times \text{product of roots}}{(1 + \text{product of roots})^2}$$

$$= \frac{(-2a)^2 - 4(2b-1)}{(1 + (2b-1))^2}$$

$$= \frac{4a^2 - 8b + 4}{4b^2}$$

$$= \frac{a^2 - 2b + 1}{b^2}$$